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Modeling Time-Varying Variance-Covariance for Exchange Rate Using Multivariate GARCH Model

"Case Study of Bivariate BEKK-GARCH Model of US Dollars and Kenyan Shillings Vis a Vis Rwandan Francs"

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Abstract: **This work contains a review of GARCH models families, where BEKK-MGARCH model is presented and applied to 4 years data of Rwanda exchange market. 1632 daily observations average of buying and selling Rwanda exchange rate in the period from Friday 12/03/2010 to Saturday 26/07/2014, to model the time varying variance-covariance in USD/Rwf, and Ksh/Rwf exchange rate series in Rwanda exchange market. Time varying variance-covariance have been constructed with the models (BEKK-GARCH) significant at lag 4 using AIC, BIC criteria and QMLE method have been used to estimate parameters. R, Eviews and RATS software have been used for data analysis. Results shows that exchange rate market responds more to shocks arriving from other exchange rate markets (usd/rwf to Ksh/Rwf and vis versa) than it does to its own shocks (usd/rwf), it confirm our hypothesis of existential of cross-market spillover effects in the studied areas. We have worked with two series (USD/RWF and KSH/RWF), while Rwanda Exchange market have also many other major currencies, we propose to future researchers to incorporate many others majors currencies like EURO/RWF, USH/RWF, as they may have an important impact on time varying variance covariance model.**

Keywords: **Exchange rate, BEKK-GARCH model, Volatilities, Forecasting.**

I. INTRODUCTION

1.1. Background of the study:

One of the greatest concerns in financial time series is the problem of "heteroskedacity", the phenomena of volatility clustering, asymmetric effect in the returns; periods are more volatile than others due to perturbation and unexpected event. In 1982 Engle developed techniques to model such heteroskedasticity and call it "Autoregressive Conditional Heteroskedasticity"(ARCH). ARCH model is popular to model volatility in financial time series. It was modified by Bolleslev in 1986 to generalized ARCH (GARCH), the model that enabled researchers to study volatility in financial time series in a univariate approach.

While modeling volatility of the returns has been the main centre of attention, understand the comovement of financial returns is of great practical importance. One of weaknesses of GARCH model is remarkable when we want estimate covariance that depends on interaction of variables, at this time GARCH will no longer be able to deal with the situation, reason why a model that take simultaneously more than one time series into account would be more convenient when estimating time varying covariance and time varying correlation. It is therefore important to extend the univariate GARCH to multivariate GARCH (MGARCH) models.

In recent years, a family of MGARCH models has been developed. The first developed is VEC. The implementation of estimation in MGARCH model with empirical data is of more complex nature since the number of parameters increase rapidly. This issues inspired researchers to propose different models with purpose of arriving at parsimonious model (with minimum number of parameters) such as BEKK, DCC (Dynamic constant correlation) and CCC (constant conditional correlation).

The exchange rate volatility is of great concern in financial time series from its impact on inflation and international trade. Furthermore it is used in risk management where it's estimation and forecasting is of great value in financial market.

In Rwandan marked, some periods have been found to be accompanied by high depreciation; in 1994 genocide was found to increase the exchange rate volatility, one of Rwandan policy of smoothing such depreciation have been put into place such us integration in EAC in 2007 that tried to reduce exchange rate volatility (William K. et al , 2013). They was a depreciation of Rwandan francs (frw) against USD; in 2012 there was a depreciation of 4.5% against USD, the National Bank tried to smoothen the rwf exchange rate volatility by selling foreign exchange to banks, the depreciation raised again to 4.9% by June 2013 (BNR annual report 2013)

The depreciation seems to increase day by day, studies have been done; William K., at al (2013) studied exchange rate volatility in Rwanda stock market. Estimation of extreme risk due to the exchange rates fluctuation in Rwanda market (Mung'atu et al, 2010), they recommended us to use asymmetry models to capture the impact of short coming to risk management.

Asymmetric information effects are found in inflation which affect the rate of output growth (Shields et al, 2005; Grier et al, 2004). An asymmetric dependence between returns implies that correlations between returns are larger during episodes of financial distress compared to periods of relative stability (Patton, 2004; Hong et al. 2004).

In this study, we have used data from Rwanda exchange market. 1632 daily observations of average of buying and selling of Rwanda exchange rate within a period from **Friday 12/03/2010** to **Saturday 26/07/2014** , to model the time varying conditional covariance and correlation feature in USD/Rwf, and Ksh/Rwf exchange rate series. Two series have been picked from others due the fact that US Dollars is the most used currency in Rwanda and Ksh is selected from its strength in EAC and it have been thought that they have a great impact on the Rwandan Market fluctuation. We will perform a model selection procedure for the **BEKK-MARCH** based on the Akaike Information criterion **AIC** (and **BIC** criterion). The proposed model will help capture asymmetric effect in the Rwanda market of exchange rate using estimated non-constant covariance and capture the time varying correlations between the USD/Rwf and Ksh/Rwf exchange rate series.

1.2. Statement of the problem:

Previous research on Rwanda exchange market (Mung'atu et al, 2010) studied the exchange risk fluctuation and raised the problem of asymmetric effects of exchange rate of major currencies. It is not intellectual to ignore the asymmetric effect that may come from interaction of two or more currencies to the fluctuation of the exchange market. Asymmetric effect information between currencies is given by the time varying variance-covariance; we would like use this model (time varying variance-covariance model) to capture the asymmetric effect between currencies.

The main purpose of this study is to estimate time varying variance-covariance model using BEKK-GARCH model to study asymmetric shocks effect on Rwanda exchange market.

1.3. Justification:

The research is beneficial to me, since it is an application of the theory studied in applied statistics, in the course of financial time series, and it shows that there is a major concern to take into consideration the asymmetric effect when doing analysis of two or more time series. It is also beneficial to future researcher who may need it as reference when studying the asymmetric effect of two or more time series.

1.4. Hypothesis:

It has been hypothesized that the asymmetric effects (volatility transmission) between USD/Rwf and Ksh/Rwf affect the fluctuation of Rwanda exchange market.

1.5. Objectives:

The general objective is to estimate asymmetric effects using time varying variance-covariance between two currencies USD/Rwf and Ksh/Rwf in Rwanda exchange market.

Specific objectives

1. To estimate a univariate model for USD/RWF currency to capture the volatility.
2. To estimate a univariate model for USD/RWF currency to capture the volatility.
3. To estimate a BEKK-MGARCH model to examine the volatility transmission (Spillover effect) in both currencies

1.7 Limitation of the study:

Our scientific work is oriented in applied statistics in the field of financial time series we explore the family of generalized autoregressive heteroscedasticity model in multivariate analysis.

II. LITERATURE REVIEW

John D., et al (**2010**) evaluated how GARCH model replicates the empirical nature of sequences of GBP/USD exchange rate, JPY/USD exchange rate and EURI/USD exchange rate via simulation.

Dahiru et al (**2013**) , in Nigeria examined exchange rate volatility with GARCH models using monthly exchange rate return series from 1985 to 2011 for Naira/USD return and from 2004 to 2011 for Naira/GBP and and Naira/EURO returns.

The study of Bollerslev (**1987**) on speculative prices and rates of returns used GARCH model, he has taken the conditional means along with GARCH model with conditional variance using data of daily spot prices from the New York foreign exchange market on the US dollars versus the British pound and the Deutschmark from March 1, 1980 until January 28, 185.

In a study conducted in Nigeria by Babatunde et al(**2010**) the ARCH and GARCH models were used to examine the degree or severity of volatility based on the first difference, standard deviation and coefficient of deviation estimated volatility series for the nominal and real exchange rate of Naira vis-a-vis the U.S dollar. The result indicated the presence of overshooting volatility shocks.

Maana et al (**2010**) applied generalized autoregressive conditional heteroscedasticity process estimating the volatility in the Kenyan exchange rates. A quasi-maximum likelihood estimation procedure is used and asymptotic properties of the estimators given. Exploratory data analysis performed indicates the returns are heavy tailed. It is found that the estimated model fits well the exchange rates return data for the period 1993-2006.

Wei, Ching-Chun, (2008) with Multivariate GARCH model they analyzed the unexpected U.S. D, Yen and Euro-dollar to Reminibi volatility spillover to stock markets. The results from DCC-MGARCH (1, 1) model showed that ARCH and GARCH effect exist. The DCC parameters were insignificantly and the sum value of the parameters were less than one, shows that model is mean reverting.

FONG PAK WING (2001) in his study on "topics in time series" developed a theory on approach of modeling subset multivariate ARCH model using AIC and BIC principals.

Angel L. et al (2005) employed a multivariate BEKK GARCH model that allows news to affect the conditional volatility in an asymmetric manner. They estimate the conditional variances and covariances of the Japanese yen, Swiss franc and British pound vis-à-vis the US dollar over a long time series from January 1971 to June 2005. They find evidence of significant spillover effects across markets which are determined by the type of news arriving in the markets. Analysing the dynamics of exchange rate volatility, they find conditional volatilities, covariances and correlations between exchange rates to be time varying.

III. METHODOLOGY

This section describes techniques and methods used in this work, it describe first univarite GARCH model and after the Multivariate GARCH models are described. Statistical tests and techniques for estimations are developed in this section.

3.1. Univariate ARCH model:

ARCH model is an Autoregressive conditional heteroschedastic, it attempts to explain variance clustering in the residuals and imply nonlinear dependence among the squared errors of the first moment model. Engle (1982) relaxes the constant conditional variance assumption in traditional Box-Jenkins ARIMA models and allows it to follow a process as below

$a_t = \sigma_t \varepsilon_t$ and $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2$

Where $\sigma_t^2 = E(a_t^2/\mathrm{F}_{t-1}) = \mathrm{Var}(a_t/\mathrm{F}_{t-1}) = \mathrm{Var}(y_t/\mathrm{F}_{t-1})$. Letting $e_t = a_t^2 - E(a_t^2\,/\,\mathrm{F}_{t-1})$ (which is the same as $e_t = a_t^2 - \sigma_t^2$), the above equation can also be written as an AR (q) model as below

$a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2 + e_t$

where e_t is a white noise process. A model with σ_t^2 is referred to as an **autoregressive conditional heteroscedastic (ARCH)** model, or ARCH (q) model. For such models, it is required that $\alpha_0 \geq 0, \alpha_1 \geq 0, \ldots \alpha_q \geq 0$ and e_t are $i.i.d.$ normal random variables with zero mean and variance one, that is, $e_t \sim NID(0,1)$.

The log likelihood function of an ARCH model, with the assumption that ε_t follows a Normal distribution is

$L(\alpha) = \sum_{t=q+1}^{n}\left[\frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}(a_t^2/\sigma_t^2)\right]$

3.2. Univariate GARCH model:

Although the ARCH model is simple, it requires many parameters to describe volatility process of an asset return. Some alternative models have been proposed; Bollerslev (1986) proposes a useful extension known as the Generalized ARCH model (GARCH).

For a log return series (also called Log price relative) r_t let $a_t = r_t - \mu_t$

Then a_t follows a $GARCH(p,q)$ model if;

$a_t = \sigma_t \varepsilon_t$, $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_i \sigma_{t-j}^2$

Where $\{\varepsilon_t\}$ is a sequence of iid random variables with mean Zero (0) and variance one (1). The $GARCH(q,p)$ model satisfies the following constraints $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j > 0$ and

$\sum_{i=0}^{\max(p,q)}(\alpha_i + \beta_j) < 1$ is less than one.

The α_i and β_j are referred to as ARCH and GARCH parameters, respectively. The unconditional σ_t^2 variance evolves over time$t$.

Therefore the above model can be explained as follows;

$$\underbrace{\sigma_t^2}_{Conditional\ variance} = \underbrace{\alpha_0}_{Intercept} + \underbrace{\sum_{i=1}^{q} \alpha_i a_{t-i}^2}_{ARCH\ effect} + \underbrace{\sum_{j=1}^{p} \beta_i \sigma_{t-j}^2}_{GARCH\ effect}$$

It is easy to see that GARCH (p, q) model reduce to ARCH (q) provided that p=0. Under the normality assumption of ε_t , the log likelihood function of the vector α of GARCH model is the same as ARCH model mentioned above.

3.3. MULTIVARIATE GARCH MODELS:

Modeling volatility returns has been the main centre of attention. Understanding the comovement of financial returns, markets interact are of great practical importance. It is therefore to extend the consideration from univariate GARCH model to Multivariate GARCH model (MGARCH).

MGARCH models can be categorized into four types according to O'Hara (1995):

1. Models of the conditional covariance matrix: The conditional covariance is computed in a direct way. For example the VEC and BEKK models.

2. Factor models: The return process is assumed to consist of a small number of unobservable heteroscedastic factors. This approach benefits from that the dimensionality of the problem reduces when the number of factors compared to .dimension of the return vector is small.

3. Models of conditional variances and correlations: At first the univariate conditional variances and correlations are computed and then used to get the conditional covariance matrix. Some models are for example the Constant Conditional Correlations (CCC) model and the Dynamic Conditional Correlations (DCC) model.

4. Nonparametric and semiparametric approaches: Models in this class form an alternative to parametric estimation of the conditional covariance structure. The advantage of these models is that they do not impose a particular structure (that can be misspecified) on the data.

In this work, we use asymmetric BEKK-MGARCH model of the conditional time varying variance-covariance model. The study is using two exchange rates series, resulting in a bivariate approach.

Consider the vector stochastic process $\{y_t\}$, we write $y_t = \mu_t + \varepsilon_t$ where μ_t is the conditional mean vector, and

$$\varepsilon_t = H_t^{1/2} Z_t$$

Where $H_t^{1/2}$ is $N \times N$ positive definite matrix, we assume Z_t , the $N \times 1$ vector to have the following first two moment:

1. $E(Z_t) = 0$
2. $Var(Z_t) = I_N$ where I_N is the identity matrix of order N.

Once we calculate the conditional variance matrix;

$$Var_{t-1}(y_t) = Var_{t-1}(y_t)$$
$$= H_{ij,t}^{1/2} Var_{t-1}(y_t) (H_{ij,t}^{1/2})'$$
$$= H_{ij,t}$$

For the bivariate case $H_{ij,t} = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}$ called conditional variance-covariance matrix. Where i=1, 2 and j=1, 2

It is clear that $H_{ij,t}^{1/2}$ is $N \times N$ positive definite matrix such that H_t is the conditional variance matrix of y_t.

A general formula of H_t have been proposed by Bollerslev (1988), in the general **VEC model**

$$h_t = c + A\eta_{t-1} + Bh_{t-1}$$

Where $h_t = vech(H_t)$ and $\eta_{t-1} = vech(\varepsilon_t \varepsilon_t')$, and $vech(.)$ denotes the operator that stack the lower triangular portion of $N \times N$ matrix as a $(\frac{N(N+1)}{2} + 1)$ vectors. A and B are square parameters matrices of order $\frac{(N+1)\text{N}}{2}$ and C is $\frac{(N+1)N}{2}$ parameters vector.

The number of parameters in the whole VEC model is $N(N+1)(N(N+1)+1)/2$; for $N = 2$ for example we will have 21 parameters, for $N = 3$ we have 78 parameters, one can understand what will be the situation when N is very high. It means that the model is used only for small N.

Because it is difficult to be sure of the positivity of H_t in VEC model representation without imposing restriction on parameters; Engle and Kroner (1995) propose a new parameterization for that easily imposes its positivity, i.e. the BEKK model (the acronym comes from synthesized work on multivariate models by Baba, Engle, Kraft and Kroner)

Let H_t be measurable with respect to previous information ($\mathcal{F}_{t-1}$); then the multivariate GARCH model can be written as

$$H_t = C'C + \sum_{i=1}^{q} A_i' \, \mathcal{E}_{t-i} \mathcal{E}_{t-i}' A_i + \sum_{j=1}^{p} B_j' H_{t-j} B_j$$

Where C, A_i and B_j are $N \times N$ parameters matrices, but C is upper triangular. The part $\sum_{i=1}^{q} A_i' \, \mathcal{E}_{t-i} \mathcal{E}_{t-i}' A_i$ of the model is the ARCH effect and the following part $\sum_{j=1}^{p} B_j' H_{t-j} B_j$ is the GARCH effect.

A bivariate GARCH (1, 1) model is presented,

$$H_{ij,t} = \begin{bmatrix} h_{11,t} & h_{12,t} \\ 0 & h_{21,t} \end{bmatrix}$$

Where univariate models are presented as follow;

$$h_{11,t} = C_{11} + a_{11}^2\varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\epsilon_{1,t-1} + a_{11}^2\varepsilon_{2,t-1}^2 + g_{11}^2 h_{11,t-1} + 2g_{11}g_{21}h_{12,t-1} + g_{21}^2 h_{22,t-1}$$

$$h_{12,t} = C_{12} + a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{21}a_{12} + a_{11}a_{22})\epsilon_{1,t-1}\epsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 + g_{11}g_{12}h_{11,t-1} + (g_{21}g_{12} + g_{11}g_{22})h_{12,t-1} + g_{21}g_{22}h_{22,t-1}$$

$$h_{22,t} = c_{22} + a_{12}^2\varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\epsilon_{1,t-1}\epsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2 + g_{12}^2 h_{11,t-1} + 2g_{12}g_{22}h_{12,t-1} + g_{22}^2 h_{22,t-1}$$

We introduce the BEKK model, in the following section, and it will help us insert the asymmetric effects in the general MGARCH model.

3.3.1. BEKK-MGARCH model:

Engle and Kroner (1995) propose a general multivariate GARCH model and call it a BEKK representation, where the restriction of a symmetrical variance-covariance structure is removed and behave in an asymmetric manner. Let $\mathcal{F}_{t-1}$ be the sigma field generated by the past values of $\mathcal{E}_t$ and let $H_t$be the conditional covariance matrix of the N −dimensional random vector$\mathcal{E}_t$.

The GARCH-BEKK specification permits the estimation of spillover effects between equations. Asymmetric effects, for a multivariate model, can be specified as follows;

Let $K_{1,t-1}$ is a 2×2 identity matrix if $\varepsilon_{1,t-1} < 0$ and $\varepsilon_{2,t-1} < 0$ or a 2×2 matrix of 0's otherwise. Similarly, let $K_{2,t-1}$ be 2×2 identity matrix if $\varepsilon_{1,t-1} < 0$, $\varepsilon_{2,t-1} < 0$, or a 2×2 matrix of zeros otherwise.

Let $\zeta_{p,t-1} = K_{p,t-1}\varepsilon_{2,t-1}$ for p = 1 in this case. For ease of exposition we assume a MGARCH (1, 1) specification (where p=q=1):

$$H_t = C'C + A\,\mathcal{E}_{t-i}\mathcal{E}'_{t-i}A' + D\zeta_{1,t-1}{}'\zeta_{1,t-1}D' + BH_{t-1}B'$$

Where

$A\,\mathcal{E}_{t-i}\mathcal{E}'_{t-i}A'$: is the ARCH effect.

$BH_{t-1}B'$: is the GARCH effect and.

$D\zeta_{1,t-1}{}'\zeta_{1,t-1}D'$: is the asymmetric effect of both series.

In the above asymmetric BEKK model; D is a matrix of coefficients for the asymmetric effects. The symmetric GARCH-BEKK model is a restricted version in which D = 0.

We let $r_{m,t}$ denote the continuously compounded returns for the US dollar /Rwandans francs rate (m=1) and the Kenyan Shilling rate (m=2).

IV. RESULTS

The data used in this study contains 1632 daily observations on two exchange rates from Friday 12 March 2010 to Saturday 26 July 2014. The exchange rates are *vis-à-vis* the Rwandan francs and the currencies are the US Dollars and Kenyan Shilling. The data are the Foreign Exchange Rate series as it have been produced by National Bank of Rwanda (BNR).

Data have been processed and analyzed in means of **R** software that helps do the description of univariate GARCH (1, 1) models for both series, **Eviews** software for determination of descriptive statistics and **RATS** statistical software that helped display the BEKK-GARCH bivariate model.

4.1. Descriptive statistics:

In this section, we emphasizes on descriptive statistics of both USD/Rwf and Ksh/Rwf series. We describe data to be used in multivariate GARCH models.

Table 1: Descriptive statistics

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
USD/Rwf	1631	622.26	68.97	691.23	621.0624	37.65842	1418.156
Ksh/Rwf	1631	2.25	5.69	7.94	7.2966	.43395	.188

Table 2: Pairwise Granger Causality Tests

Sample: 3/12/2010 7/26/2014
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Probability
KSH/Rwf does not Granger Cause USD/Rwf	1632	2.81015	0.06049
USD/Rwf does not Granger Cause KSH/Rwf		10.8433	2.1E-05

The alternative hypothesis being that the given currency granger cause the other currency,

The probabilities given by the software doesn't allow us the rejection of the null hypothesis at lag (2). Therefore given currencies doesn't granger cause each others.

Table 6: CONSTANT CORRELATION BETWEEN USD/Rwf and Ksh/Rwf

	USD	KSH
USD	1	0.63265264238
KSH	0.63265264238	1

The constant correlation have been computed to see the relationship between both series and a correlation estimated at 63% is found, this strength our ideas of moving forward and examine the time varying variance-covariance for asymmetric effects.

Both series have been graphed using R software and they have been displayed as follows

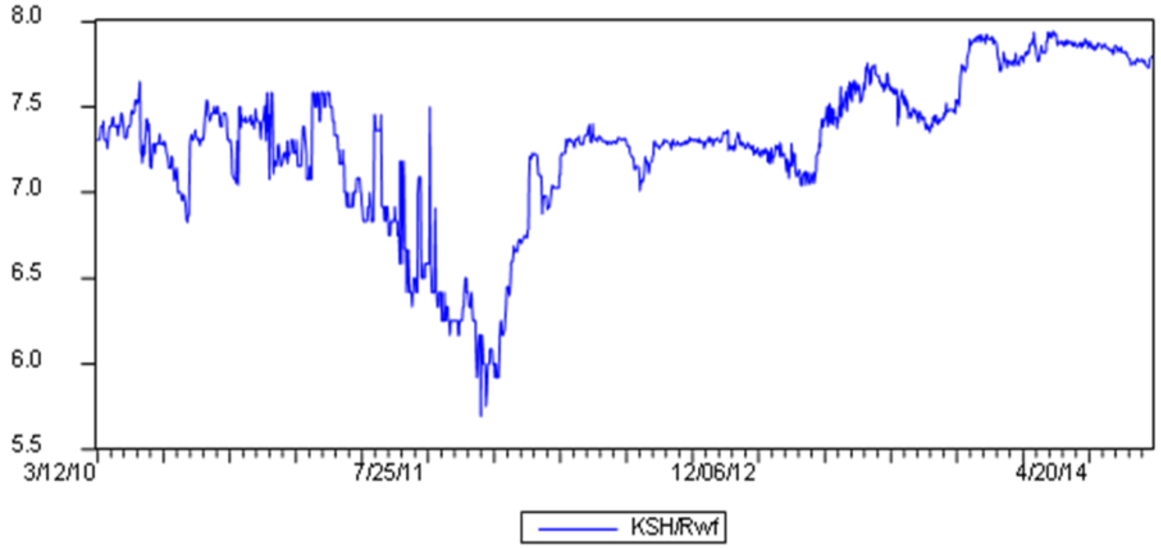

Figure 1: exchange rate of Kenyan shilling vis à vis Rwandan francs

Figure 1: shows the evolution of the exchange rates over time. Generally speaking, the Ksh depreciates followed by collapse of the fixed exchange rate system in 2012 until the early 2013. Short sharp Ksh depreciation takes place from the late 2013 to 2014.

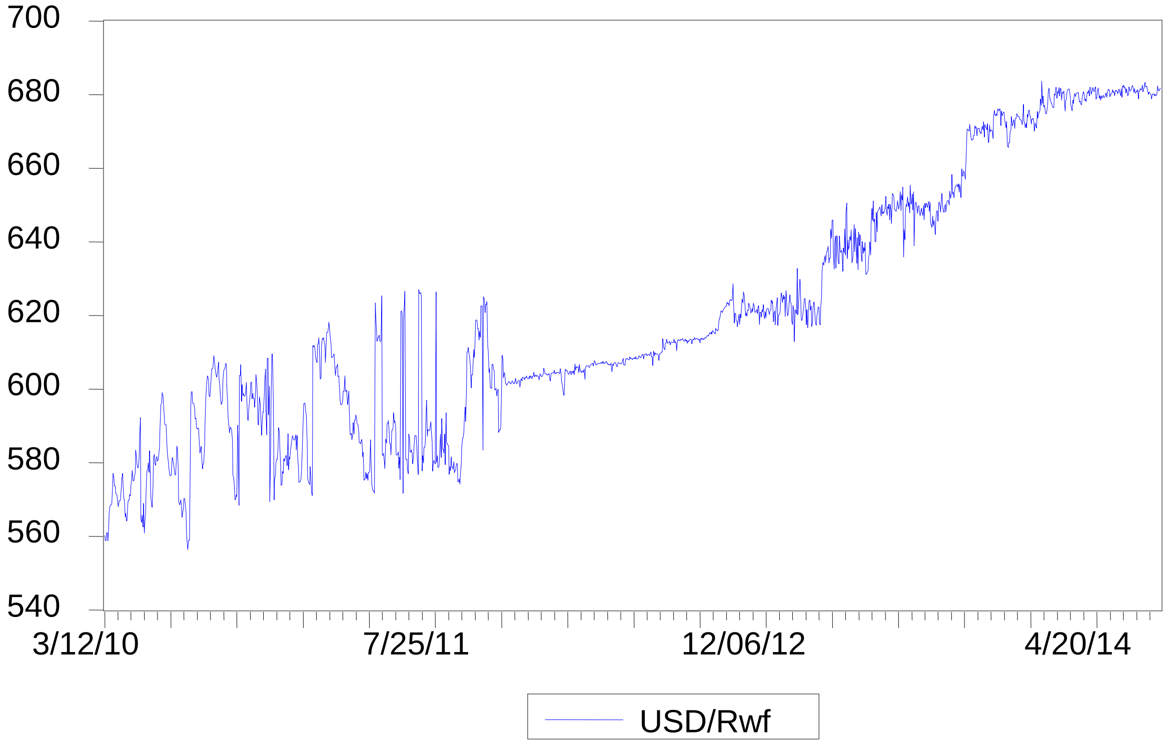

Figure 2: exchange rate of US Dollars vis à vis Rwandan francs

The high fluctuation of USD in 2010 and early 2011 followed by fixed exchange rate system in 2012 and then a short sharp fluctuation of USD/RWF exchange rate till 2014.

Both figure show that series are not stationary, no seasonality in both series, we see that there is a trend in USD/Rwf series. That is US Dollars have grown up from the first day of observation to the last day of observation.

The fact that both series are not station, we will work with the " Log Return" of every series quoted as "**Log price relative**" (LPR) in this work and it is given by:

$$LPR = r_t = \log\left(\frac{y_t}{y_{t-1}}\right) = \log(y_t) - \log(y_{t-1})$$

The LPR have been computed and plotted for both USD/Rwf and Ksh/Rwf series (LPRUSD.RWF and LPRUSD.RWF) as follows;

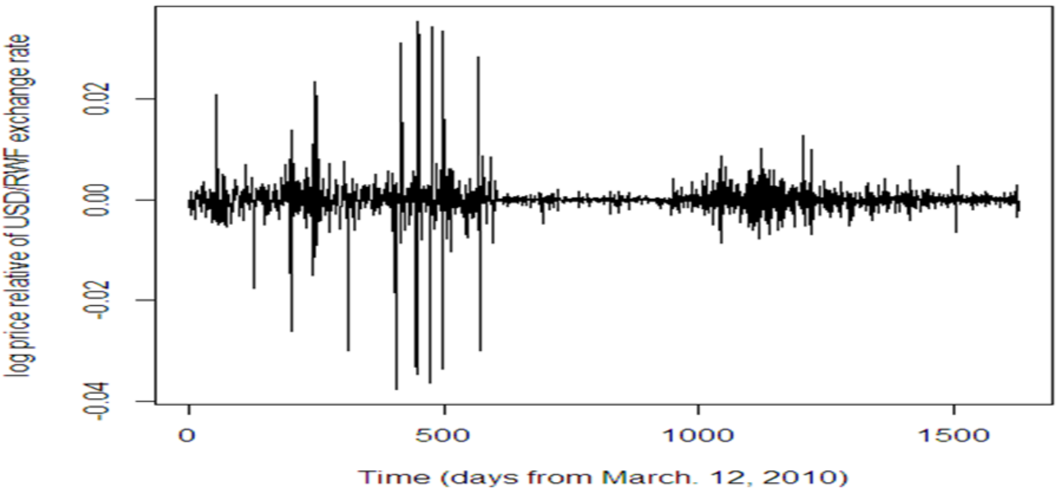

Figure 3: Log price relative of US Dollars /Rwandan Francs exchange rate

Figure 3 and 5 shows that there is volatility clustering since high pick tends to be followed by high pick and short fluctuation tends to be followed by short fluctuation. The two figure(3 and 4) and the corresponding histogram (figure 5 and figure 6) shows that the log price relative (or rate of change) of usd/rwf and ksh/rwf are stationary.

The corresponding histogram is as follows

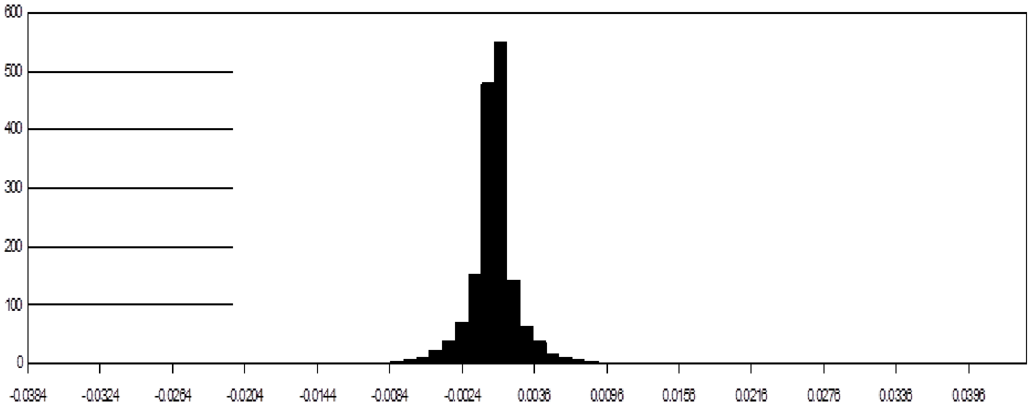

Figure 4: Histogram of Log price relative of US Dollars /Rwandan Francs exchange rate

The Log price relative of Kenyan Shilling and Rwandan francs exchange rate have been drawn as follows;

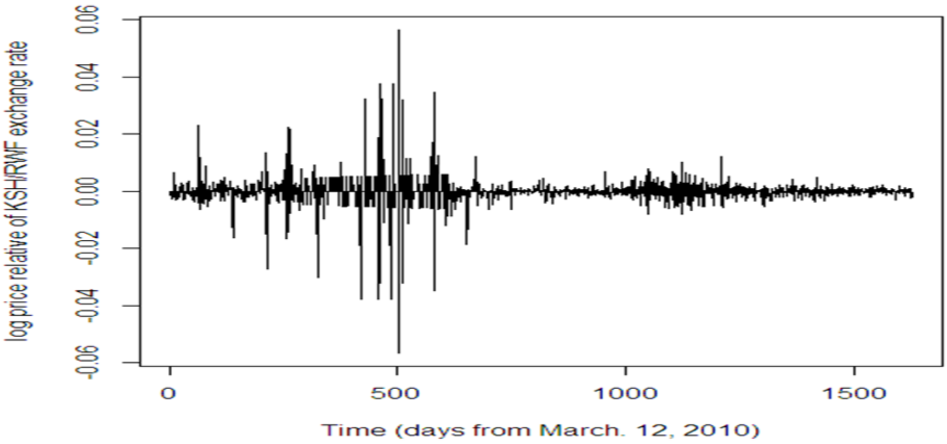

Figure 5: Log price relative of Ksh /Rwandan exchange rate

Every series shows an evidence of unpredictability and volatility clustering.

The histogram of log price relative of Ksh vis a vis Rwandan francs is as follows:

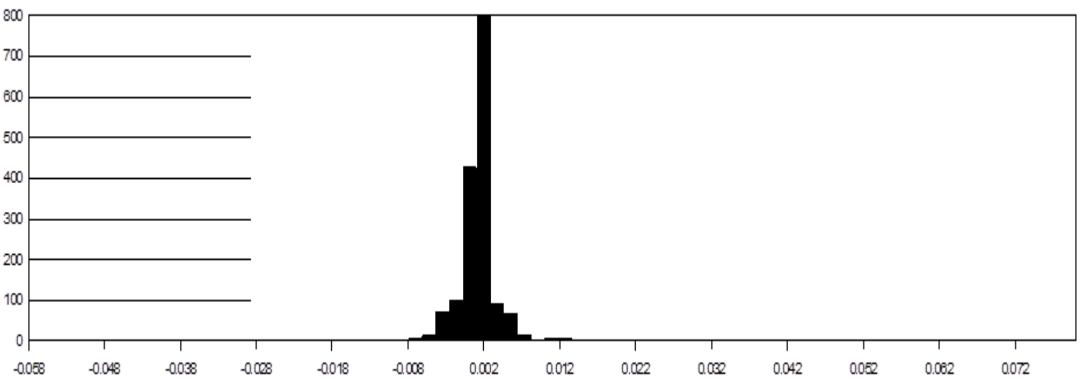

Figure 6: Histogram log price relative of Ksh /Rwandan exchange rate

Both series have also been compared in one diagram using standards values in the following figures;

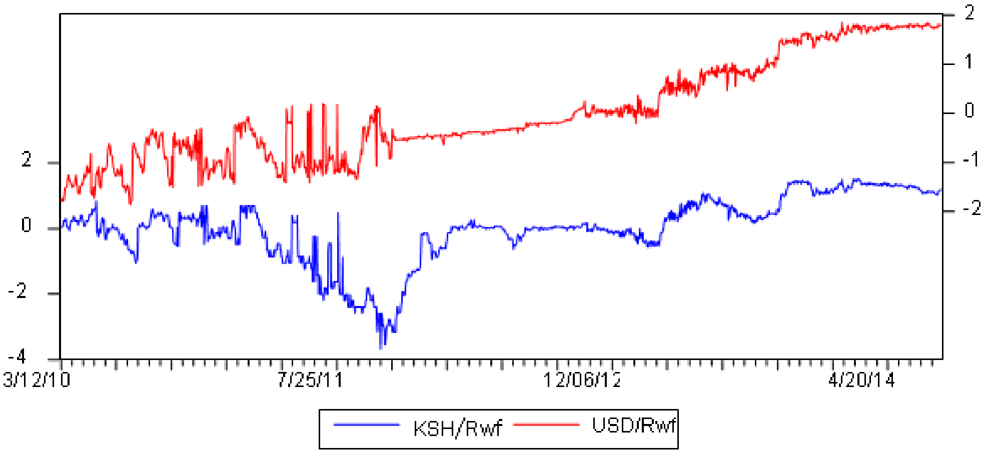

Figure 7: Usd/rwf and Ksh/Rwf series graphs using standard values

Table3 plot both series using standard values, it is seen that there is a trend in USD/Rwf and the Ksh/Rwf knew a high depreciation in 2011.

AUTOCORRELATION FUNCTION (ACF and PCF)

For the confirmation of the selected order of models, we will also determine the order of the mentioned models using the **autocorrelation function** (ACF) and **Partial autocorrelation**

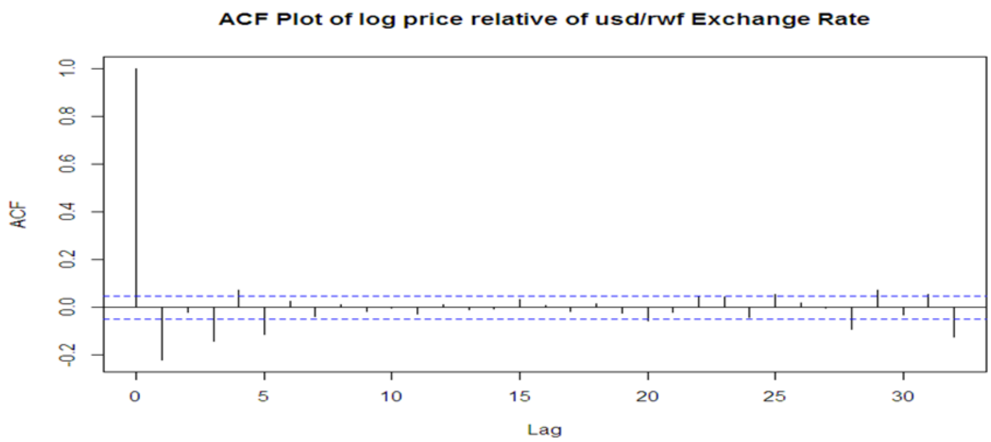

Figure 8: ACF plot of log price relative of USD/RWF exchange rate

The ACF plot of log price relative of USD/Rwf exchange rate shows insignificance of autocorrelation.

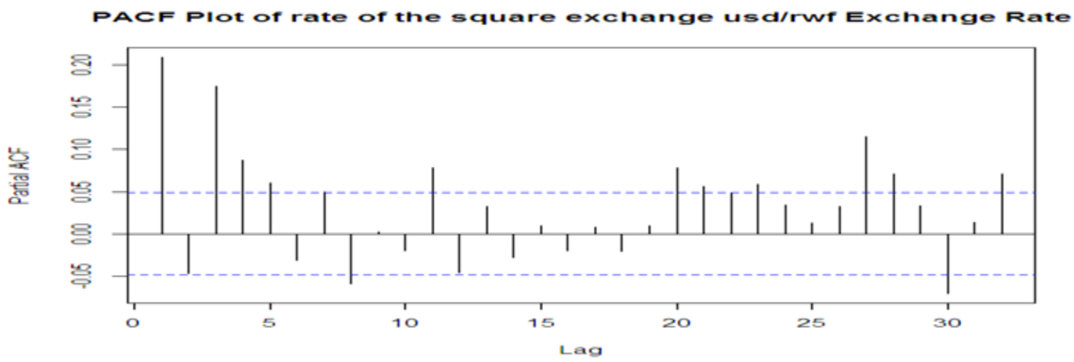

Figure 9: PACF plot of the square log relative of USD/RWF exchange rate

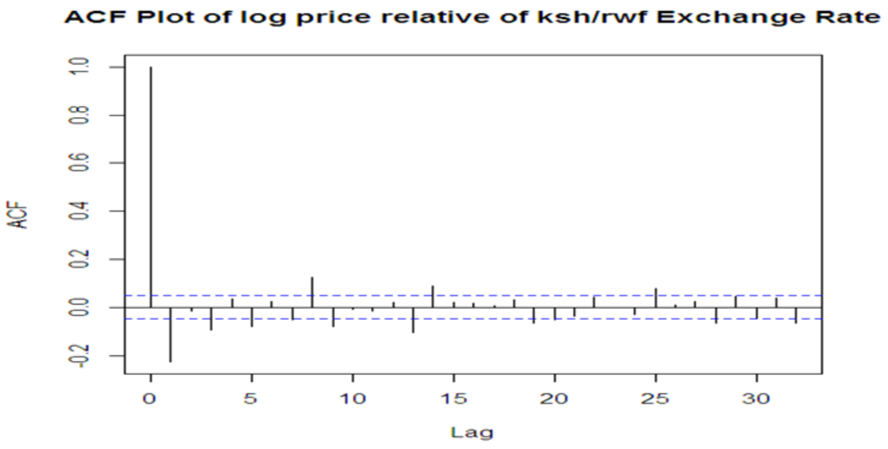

Figure 10: ACF plot of log relative of KSH/RWF exchange rate

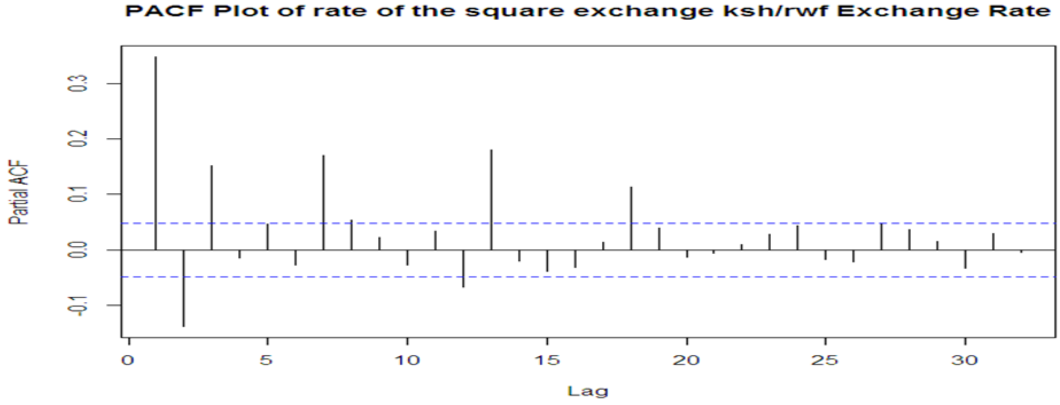

Figure 11: PACF plot of the square log relative of KSH/RWF exchange rate

The plotted ACFs show insignificant autocorrelation in two series.

4.2. GARCH MODEL:

LPR of USD/Rwf

We have fitted the GARCH (1, 1) model using the LPR of USD/Rwf using R software and the following summary has been found;

Table 3: Coefficients of GARCH (1, 1) model of USD vis à vis Rwf

	Estimate	Std. Error	t value	Pr(>|t|)
a_0	7.349e-09	2.039e-09	3.604	0.000313
a_1	2.725e-01	6.705e-03	40.648	2e-16
b_1	8.408e-01	2.650e-03	317.323	2e-16

The fitted model is written as follows

$$garch(1,1):\ \sigma_t^2 = 0.000000007349 + 0.02725a_t^2 + 0.08408\sigma_{t-1}^2$$

All coefficients are significant according to values from above table

Diagnostic Tests:

Jarque Bera Test

data: Residuals

X-squared = 36241.86, df = 2, p-value < 2.2e-1

Box-Ljung test

Data : Squared. Residuals

X-squared = 0.0887, df = 1, p-value = 0.7658

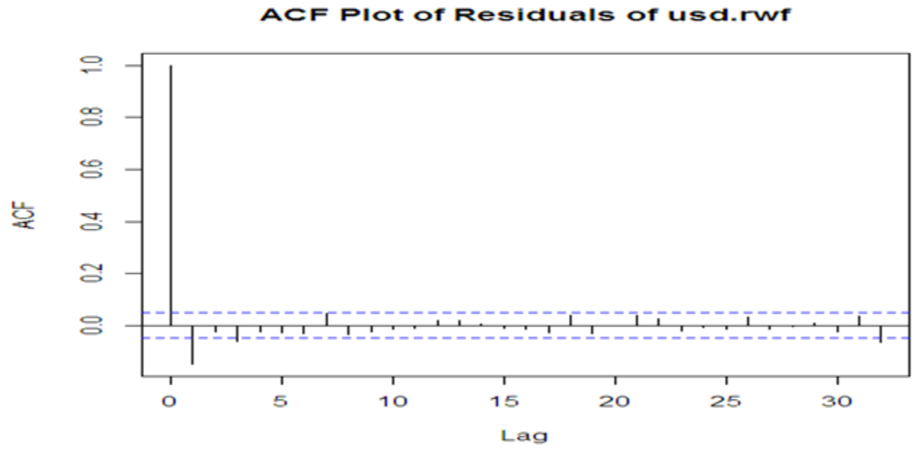

Figure 12: ACF plot of residual of LPR of USD/RWF exchange rate

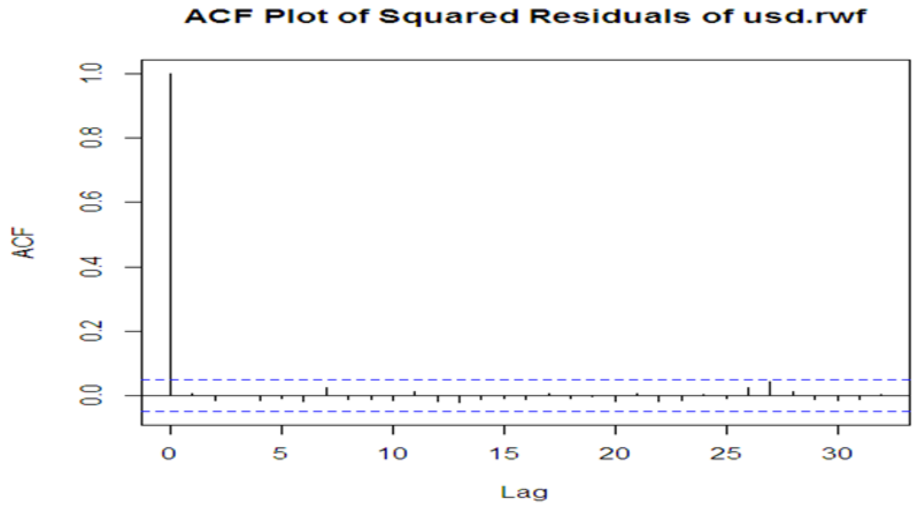

Figure 13: ACF plot of the squared residual of LPR of USD/RWF exchange rate

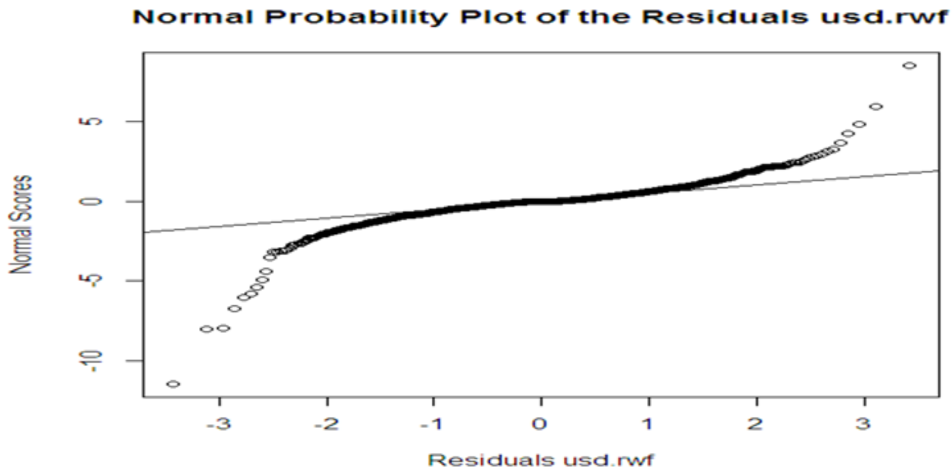

Figure 14: Normal probability plot of residual of LPR of USD/RWF exchange rate

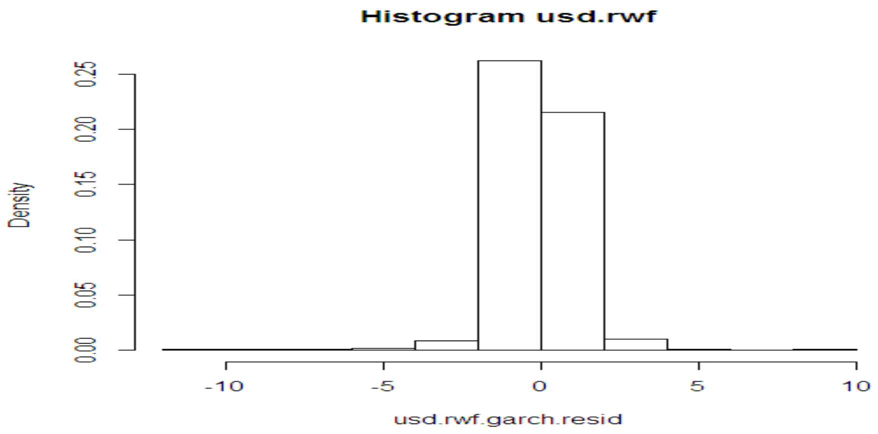

Figure 15: Histogram of residual of LPR of USD/RWF exchange rate

GARCH Model of LPR of KSH/Rwf

We have fitted the GARCH (1, 1) model using the LPR of Ksh/Rwf using R software and the following summary have been found

Table 4: Coefficients of GARCH (1, 1) model of USD vis à vis Rwf

Coefficients	Estimate	Std. Error	t value	Pr(>|t|)
$\boldsymbol{a_0}$	**2.964e-08**	4.413e-09	6.717	1.86e-11
$\boldsymbol{a_1}$	**1.581e-01**	4.072e-03	38.810	2e-16
$\boldsymbol{b_1}$	**8.921e-01**	1.950e-03	457.487	2e-16

From the table we write the following GARCH(1, 1) model;

$$garch(1,1){:}\ \sigma_t^2 = 0.00000002964 + 0.1581a_t^2 + 0.8921\sigma_{t-1}^2$$

Diagnostic Tests:

Jarque Bera Test

data: Residuals

X-squared = 39003.85, df = 2, p-value < 2.2e-16

Box-Ljung test

data: Squared. Residuals

X-squared = 0.1495, df = 1, p-value = 0.699

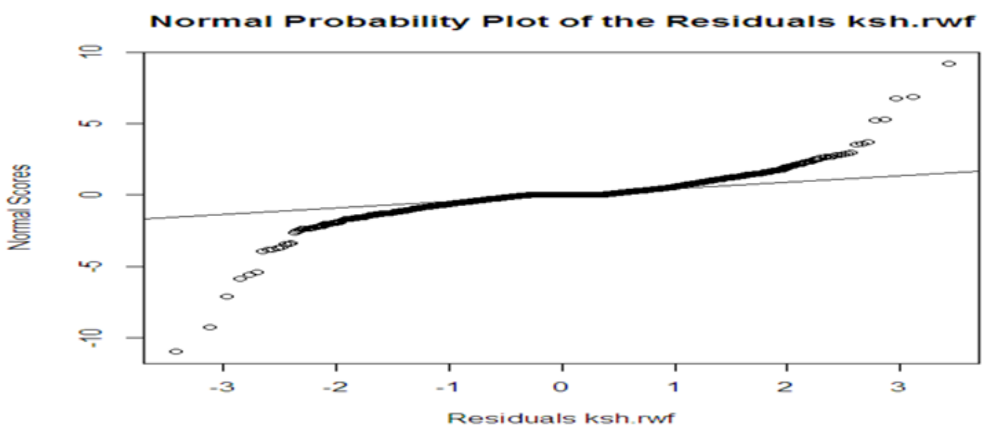

Figure 16: Normal probability plot of residual of LPR of KSH/RWF exchange rate

The above two normal probability plots (output of R software) of residuals of both series show that residuals are normally distributed.

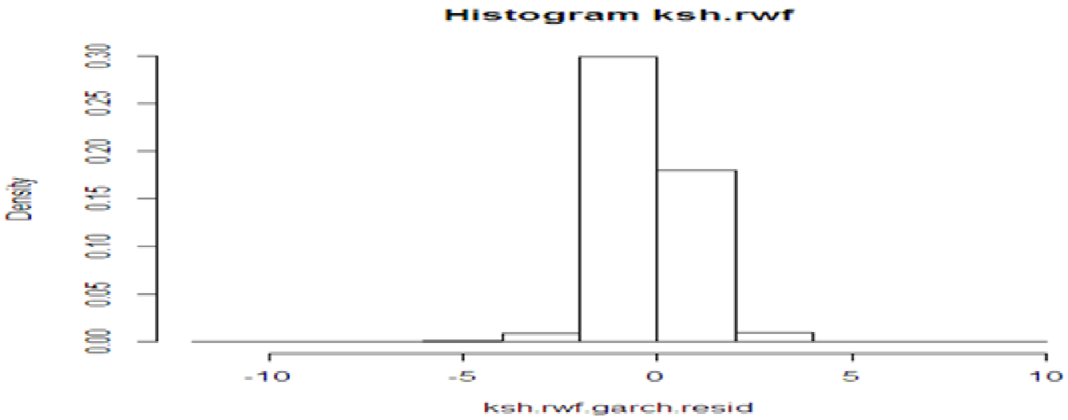

Figure 17: Histogram of residual of LPR of KSH/RWF exchange

4.3. BIVARIATE BEKK-GARCH:

GARCH (P=1,Q=1,MV=BEKK,ASYMMETRIC) with LPR(USD/RWF) and LPR(KSH/RWF)

To investigate the asymmetric effect between both series we have estimated the asymmetric time varying variance-covariance with **BEKK** model using **RATS** software (version 8.3) and the outcomes are the following:

Table 5: MV-GARCH, BEKK – Estimation by BFGS

	Variable	Coefficient	Std Error	T-Stat	Significance
1	Mean(1)	-0.000105056	0.000037413	-2.80800	0.00498501
2	Mean(2)	-0.000011911	0.000039876	-0.29870	0.76516854
3	C(1,1)	0.000163143	0.000065395	2.49472	0.01260553
4	C(1,2)	0.000329118	0.000035990	9.14472	0.00000000
5	C(2,2)	-0.000000002	0.000162901	-1.37604e-005	0.99998902
6	A(1,1)	0.438739924	0.027265115	16.09162	0.00000000
7	A(1,2)	-0.160987380	0.027979012	-5.75386	0.00000001
8	A(2,1)	0.018317096	0.022341727	0.81986	0.41229573
9	A(2,2)	0.197406614	0.018566800	10.63224	0.00000000
10	B(1,1)	0.430507753	0.021542639	19.98398	0.00000000
11	B(1,2)	0.880798662	0.022351312	39.40702	0.00000000
12	B(2,1)	-0.876512839	0.023605402	-37.13188	0.00000000
13	B(2,2)	0.214389109	0.023150006	9.26087	0.00000000
14	D(1,1)	-0.268880013	0.042916824	-6.26514	0.00000000
15	D(1,2)	-0.258515198	0.036188983	-7.14348	0.00000000
16	D(2,1)	0.287283387	0.030647951	9.37366	0.00000000
17	D(2,2)	0.148469757	0.032114751	4.62310	0.00000378

We can see from above table (table5) that most of the variables are significant. It means that they are ready to be used in BEKK-model. Matrices A, B, C and D can also be written using matrix notation, if we take an example of matrix $D_{2\times 2}$, which explain more on our interests, can be written as follows:

$$D_{2\times 2} = \begin{bmatrix} -0.268880013 & -0.258515198 \\ 0.287283387 & 0.148469757 \end{bmatrix}$$

4.4. Dickey-Fuller Unit Root Test:

AKAIKE INFORMATION CRITERIA

There are several information criteria available to determine the order of model process. All of them are likelihood based. The well-known Akaike information criterion (AIC) (Akaike, 1973) is defined as:

$$AIC = \frac{-2}{N}\ln(likelihood) + \frac{2}{N} \times (number\ of\ parameters)$$

Where the likelihood function is evaluated at the maximum likelihood estimates

and N is the sample size.

Another information criteria is the Bayesian Information criteria given by

$$BIC = -2(LLF) + m \times ln(N)$$

Where LLF means Log likelihood function. We will use both criteria since (AIC and BIC) since BIC is consistent but inefficient and the AIC is efficient but inconsistent. No criteria is superior the other, but an overall assessment is needed based on the results showed by the criteria. (Brooks, 2008, p. 233ff). The BIC perform better in simple models while AIC perform much better in complex models, the performance get better when the sample size is large.

We have used both AIC and BIC for lag selection, the following are **RATS** software output of Dickey-Fuller Unit roots test;

Stating Null Hypothesis as follows:

H_o: no stationality (the existence of unit root)

H_1: Existence of stationality (No unit root)

RATS output for Dickey-Fuller Unit roots test

Table 6: Dickey-Fuller Unit roots test

Regression runs from 8 to 1632
Observations 1625
With intercept

With 6 lags chosen from 6 by **AIC**

Sig Level	Critical Value
1%(**)	-3.4372
5%(*)	-2.8638
10%	-2.5680
T-Statistic -19.4166*	

With 4 lags chosen from 6 by **BIC**

Sig Level	Critical Value
1%(**)	-3.4372
5%(*)	-2.8638
10%	-2.5680
T-Statistic 22.7464* **(T-Statistic -50.4058**)**	

The test is based on the T-statistic at the selected lag is 4 (since it the min (4, 6) =4), we use the calculated **T=22.7464** comparing to tabulated **Critical value -2.8638** at 5% level of significant.

Decision: Since the calculated value is greater than the tabulated value we reject the null hypothesis of no stationality and adopt the alternative hypothesis that the data are stationary in mean and in variance.

4.5. Empirical Results:

4.5.1. Diagnostic Tests of Model:

We estimate the standard BEKK and asymmetric BEKK models using a GARCH(1,1) approach. The number of optimal lags to be used in the returns model was determined by the Schwartz Information Criterion. The BFGS (Broyden, Fletcher, Goldfarb, and Shanno) algorithm is used to maximize the log likelihood function. We adopt the quasimaximum likelihood estimation (QML) of Bollerslev and Wooldridge (1992), which allows inference when the conditional distribution of the residuals is non-normal.

Convergence is achieved is 8 iterations and 100 iterations for the standard and asymmetric models, respectively.

The selection of the preferred model specification is based on a likelihood ratio test of the **null hypothesis** that $A(m,n)$= $B(m,n)$= $D(m,n)$= 0. The likelihood test statistic equals 15004.7073, which exceeds the critical value of 104.2 from the $\chi^2_{0.005}$ distribution. This implies the **null hypothesis is strongly rejected** by the data and suggests that modeling asymmetric shocks is important in predicting the volatility of exchange rates (log price relative), which could yield an advantage to market participants.

If the model is correctly specified, the standardized residuals will be iid standard normal variables. Univariate tests are applied independently to each series. We follow the former approach and carry out independent residual diagnostic tests using the Ljung-Box test and Jarque Bera test (see Engle, 1982). The model adequately captures all of the persistence in the variance of returns since the standardised squared residuals are serially uncorrelated.

4.5.2. Volatility transmission and the effect of asymmetric shock:

The BEKK model shows the persistence of volatility following innovations in the returns. Markets respond to their own shocks and shocks from other markets. The latter is known as **spillover effects** or volatility transmission in the literature. The BEKK model estimates coefficients which quantify the impact on the conditional variance of a currency of shocks originating in that particular currency market as well as other currency markets. The estimated coefficients for the asymmetric BEKK model are shown in Table 5. Our first task is to discuss the coefficients of matrix A which are denoted by$A(m,n)$. These are coefficients on the lagged squared error terms which provide the innovations in each market (ARCH effect). Shock from each individual currency market is considerably more important in predicting foreign exchange volatility than shocks arriving about other currencies. The coefficients A (1, 1) and A(2,2), quantify the impact of shock in the usd/rwf and in Ksh/rwf currencies, respectively and coefficients are 0.438 (for Usd/Rwf) and 0.19 (for Ksh/Rwf) and highly significant. There is evidence of some significant spillover effects.

It would appear that the conditional variance of returns in the conditional volatility of the USD is also responsive to shocks about the Ksh (A (1, 2)) but Ksh doesn't affect USD (see A(2,1)).

The coefficients in matrix B indicate the persistence of shock or the rate at which shock decays. The level of shock persistence in a market is greater when shock emanates from that market. The coefficients B (1, 1) and B (2, 2) are small than the cross-market Coefficients (B (1,2) and B(2,1)). The coefficients imply that the effect of the arrival of shock from the own market doesn't lasts for at least one day with the stronger persistence. Shocks about USD and KSH decay for at least one day in the Rwf market.

Matrix D contains a set of coefficients which quantify the effect on conditional volatility of innovations in shock on days when returns to each currency are negative. Several interesting features emerge. Shock in the USD market about a depreciating Rwf lowers the variance of returns on the Ksh (D(1, 1) whereas shock in the Ksh market significantly increase the variance the log price relative on the USD (D(2,2)). The magnitude of the spillover effects in matrix D is much larger than in matrix A.

V. CONCLUSION AND RECOMMENDATION

5.1. CONCLUSION:

In this work, we used a multivariate asymmetric BEKK GARCH model to estimate the conditional volatility of log price relative (exchange rate returns) of USD/Rwf and Ksh/Rwf exchange rate between Friday 12 March 2010 and Saturday July 2014. The asymmetric model have been chosen due to the fact that markets do respond differently to good and bad shocks, and that asymmetries should be specified if market participants are to make efficient financial decisions. An exchange rate market responds more to shocks arriving from other exchange rate markets than it does to its own shocks, it shows an evidence of cross-market spillover effects. We have worked with two series (USD/RWF and KSH/RWF), while Rwanda Exchange market have also many other major currencies, we propose to future researchers to incorporate many others currencies like EURO/RWF, USH/RWF in the BEKK model as they may have an important impact on time varying variance covariance model.

5.2. RECOMMENDATION:

To researchers:

We recommend to researcher who use in their works the GARCH families in financial time series analysis, not to ignore the asymmetric effect that give rise to volatility transmission, since it have a great impact on markets under investigations.

To policy makers:

1. To base their decision on output of scientific works, so that they may have sufficient information on the situation.
2. To use complex models that help them have sustainable plan for the future using those model in forecasting.

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